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Calibration of Pressure Balances

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Calibration Guide

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CALIBRATION OF PRESSURE BALANCES

Purpose

This document has been produced to enhance the equivalence and mutual recognition of calibration results obtained by laboratories performing calibrations of pressure balances.

Authorship and Imprint

This document was developed by the EURAMET e.V., Technical Committee for Mass.

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Guidance Publications

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Contents

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Guidelines on the Calibration of Pressure Balances

1 Scope

1.1 This guideline describes calibration methods for pressure balances including an example of an uncertainty estimation for the use of a pressure balance. It applies to both gas-operated and liquid-operated pressure balances. In both cases the method is a comparative one. When the reference standard is also a pressure balance, the comparison is carried out using the cross-floating method described in this document.

1.2 Two calibration methods are described:

- a first method where the calibration determines the pressure generated by a piston-cylinder assembly under specified conditions.
- a second method where the calibration determines the mass of the piston and of the weights of the balance, and determines the effective area of the piston-cylinder assembly.

1.3 The document does not cover other methods such as the determination of the effective area from dimensional measurement, but does not preclude their use when applicable.

1.4 This document is a guideline suggesting a procedure which applies to pressure balances comprising a piston-cylinder assembly or a floating ball. It applies to industrial pressure balances using direct loading of the piston or the ball, excluding dividing or multiplying devices, and digital piston manometers. The relevant types of pressure balances typically cover the ranges:

- 1.5 kPa to 7 MPa in absolute mode and 1.5 kPa to 100 MPa in gauge mode, for gas-operated pressure balances;
- 0.1 MPa to 500 MPa in gauge mode, for liquid-operated pressure balances.

2 Range of application

The balances may be used for the calibration of any type of instrument used for pressure measurements. They can also be used for calibrating other pressure balances by the cross-floating method.

3 The principle of the pressure balance

3.1 A pressure balance consists of a vertical piston freely rotating within a cylinder. The two elements of well-machined quality define a surface called the 'effective area'. The pressure to be measured is applied to the base of the piston, creating an upward vertical force. This force is equilibrated by the gravitational downward force due to masses submitted to the local gravity and placed on the top of the piston. The piston is a part of the load.

3.2 Sometimes, for practical reasons, and essentially at low pressure, the cylinder rotates instead of the piston. The principle and the test methods are exactly the same in this case.

3.3 The pressure is transmitted to the movable element by a fluid which might be a gas (usually dry nitrogen) or a liquid (usually oil).

3.4 Sometimes the measuring element is not a piston-cylinder assembly, as in the case of the floating-ball balance which combines a ball to receive the load and a hemispheric base to support the ball. In this case a flow regulator controls the flow rate of gas in the clearance of the system. This type of pressure balance is used only for gas in gauge mode measurement.

3.5 When the masses are submitted to vacuum, the balance measures an absolute pressure. The residual pressure in the bell jar around the masses creates a force in opposition to the measured pressure. The residual pressure has to be measured and added to the pressure created by the masses.

3.6 When the overall masses are submitted to the atmosphere which also applies to the top of the piston, the balance measures a gauge pressure. In some cases, an adaptor allows the reversal of the piston-cylinder mounting: the balance then measures negative gauge pressure (below atmospheric pressure) and generates an upward force opposed to the gravitational one.

3.7 The general definition of the pressure measured by the balance is obtained by analysing the different components of the forces applied to the system. For the gas-operated balance in gauge mode, the pressure definition is as follows:

$$p_e = \frac{\sum_i m_i \cdot g \cdot (1 - \rho_a / \rho_{m_i})}{A_p \cdot [1 + (\alpha_p + \alpha_c) \cdot (t - t_r)]} \tag{3.1}$$

where:

p_e is the gauge pressure measured at the bottom of the piston,

m_i is the individual mass value of each weight applied on the piston, including all floating elements,

g is the local gravity,

ρ_a is the density of air,

ρ_{mi} is the density of each weight,

A_p is the effective area of the piston-cylinder assembly at a reference temperature t_r and at pressure p_e. Depending on the type and range of the balance, A_p can be expressed:

(a) as a constant A_0 equal to the mean value of all the determinations

(b) from the effective area at null pressure A_0 and the first-order pressure distortion coefficient λ:

$$A_p = A_0 \cdot (1 + \lambda \cdot p) ,$$

where p is an approximate value of the measured pressure p_e. It may be the nominal value.

(c) eventually, from a second-order polynomial, λ' being the second-order pressure distortion coefficient:

$$A_p = A_0 \cdot (1 + \lambda \cdot p + \lambda' \cdot p^2) .$$

α_p is the linear thermal expansion coefficient of the piston,

α_c is the linear thermal expansion coefficient of the cylinder,

t is the measured temperature of the piston-cylinder assembly during its use,

t_r is the reference temperature of the piston-cylinder assembly (usually 20 °C).

Alternatively, if the masses of the weights applied to the piston are conventional masses, the pressure is defined by the following equation:

$$p_e = \frac{\sum_i m_{ci} \cdot g \cdot \left(1 - \frac{\rho_{0a}}{\rho_0} + \frac{\rho_{0a} - \rho_a}{\rho_{m_i}}\right)}{A_p \cdot \left[1 + (\alpha_p + \alpha_c) \cdot (t - t_r)\right]} , \tag{3.1a}$$

where:

m_{ci} is the individual conventional mass value of each weight applied on the piston, including all floating elements,

ρ_{0a} is the conventional value of the air density, ρ_{0a} = 1.2 kg/m^3,

ρ_0 is the conventional value of the mass density, ρ_0 = 8000 kg/m^3,

and all other quantities as defined before.

If for all quantities SI units are used without prefixes, p_e will emerge in pascals.

3.8 For the liquid-operated pressure balance, a similar expression could be considered, and the force due to the surface tension of the liquid has to be added to the gravitational force:

$$p_e = \frac{\sum_i m_i \cdot g \cdot (1 - \rho_a / \rho_{m_i}) + \sigma \cdot c}{A_p \cdot \left[1 + (\alpha_p + \alpha_c) \cdot (t - t_r)\right]} \tag{3.2}$$

where

σ is the surface tension of the liquid,

c is the circumference of the piston or its extension at the level where it emerges from the oil.

Note: In some types of pressure balances, such as the dual-range ones, a correction has to be applied to take into account the fluid buoyancy on the piston. The value of this correction can often be higher than that due to the surface tension.

If the masses of the weights applied to the piston are conventional masses, the pressure is defined by this equation:

$$p_{\mathrm{e}} = \frac{\sum_i m_{\mathrm{c}i} \cdot g \cdot \left(1 - \frac{\rho_{0\mathrm{a}}}{\rho_0} + \frac{\rho_{0\mathrm{a}} - \rho_{\mathrm{a}}}{\rho_{m_i}}\right) + \sigma \cdot c}{A_{\mathrm{p}} \cdot \left[1 + (\alpha_{\mathrm{p}} + \alpha_{\mathrm{c}}) \cdot (t - t_{\mathrm{r}})\right]} \tag{3.2a}$$

3.9 For gas-operated absolute mode pressure balances, the measured pressure is expressed as:

$$p_{\mathrm{abs}} = \frac{\sum_i m_i \cdot g}{A_{\mathrm{p}} \cdot \left[1 + (\alpha_{\mathrm{p}} + \alpha_{\mathrm{c}}) \cdot (t - t_{\mathrm{r}})\right]} + \mu \tag{3.3}$$

where

p_{abs} is the absolute pressure measured at the bottom of the piston,

μ is the residual pressure surrounding the weights,

m_i is the individual mass value of the weights applied to the unit, referring to the mass-density and not to any conventional density.

3.10 The bottom of the piston when the balance is in equilibrium is usually considered to be the reference level of the balance. In some cases, for practical reasons, the initial weight is adjusted by the manufacturer to refer the reference level to the output connection of the balance. Special attention will be paid to the method used for the calibration of this type of instrument.

3.11 With the reference level being chosen at the bottom of the piston, equations (3.1, 3.1a, 3.2, 3.2a and 3.3) are only valid if the piston surface contacting with the pressure fluid has a simple cylindrical shape. If the piston surface deviates from the simple cylindrical shape, e.g. typically due to a free volume, a conical end or a step on the piston, additional volume V produced by this shape deviation must be taken into account for fluid buoyancy on the piston. The pressure corrected for the piston buoyancy is given by equations

in gauge mode: $$p_{\mathrm{e}\_V} = p_{\mathrm{e}} + \frac{(\rho_{\mathrm{f}} - \rho_{\mathrm{a}}) \cdot g \cdot V}{A_{\mathrm{p}} \cdot \left[1 + (\alpha_{\mathrm{p}} + \alpha_{\mathrm{c}}) \cdot (t - t_{\mathrm{r}})\right]}, \tag{3.4}$$

in absolute mode: $$p_{\mathrm{abs}\_V} = p_{\mathrm{abs}} + \frac{\rho_{\mathrm{f}} \cdot g \cdot V}{A_{\mathrm{p}} \cdot \left[1 + (\alpha_{\mathrm{p}} + \alpha_{\mathrm{c}}) \cdot (t - t_{\mathrm{r}})\right]}, \tag{3.5}$$

where ρ_{f} is the density of the measuring fluid. The additional volume V can be positive (e.g. piston with free volume or conical end) or negative (e.g. stepped piston with an increase in radius). The additional volume is typically present in the following types of pressure balances: gas-operated low range, gas-operated oil-lubricated and dual-range ones.

3.12 When the pressure is expressed at a level different from the reference level, a corrective term (the head correction) has to be added to the pressure expressed above by equations (3.1-3.5)

in gauge mode: $$p_{\mathrm{e}\_\Delta h} = p_{\mathrm{e}\_\Delta V} + (\rho_{\mathrm{f}} - \rho_{\mathrm{a}}) \cdot g \cdot \Delta h, \tag{3.6}$$

in absolute mode: $$p_{\mathrm{abs}\_\Delta h} = p_{\mathrm{abs}\_\Delta V} + \rho_{\mathrm{f}} \cdot g \cdot \Delta h, \tag{3.7}$$

with Δh being the difference between the altitude h_1 of the balance reference level and the altitude h_2 of the point where the pressure has to be measured, $\Delta h = h_1 - h_2$.

3.13 Equations (3.1-3.7) are valid for pressure balances of a floating-cylinder configuration as well. If the pressure reference level is chosen at the piston top located inside the cylinder cavity, the additional volume V in equations (3.4 and 3.5) is the volume of the cylinder cavity minus the volume of the piston part placed inside the cylinder cavity. This additional volume V is always positive.

Note: For a floating-piston as well as for a floating-cylinder configuration, the pressure reference level can always be chosen in such a way that the additional volume in equations (3.4 and 3.5) becomes equal to zero.

4 Preparing for calibration

The calibration should only be carried out when the pressure balance is in good working order. The operation of the pressure balance under calibration and the pressure reference standard should be carried out according to the laboratory's calibration procedure and the manufacturer's technical manual.

4.1 Calibration room

4.1.1 The following parameters shall be controlled according to the uncertainty regime. Typically:

- Ambient temperature within 15 °C and 25 °C, stabilised within ±2 °C. For lower uncertainty, typically 0.01 %, the temperature of the piston-cylinder assembly should preferably be measured.
- Control the opening of doors and the movement of operators to keep a stable atmosphere, and control ventilation in order to prevent intense air flow above or below the piston balances.

4.2 Device installation

4.2.1
- Install the devices away from the air disturbances such as ventilation and air-conditioning.
- Install the balance to be calibrated as near to the standard instrument as possible.
- Use a rigid, stable table supporting the full load, with its horizontal plane checked with a spirit level.
- Minimise the height difference between the reference levels of the two instruments to be compared.
- Respect the verticality of the piston as recommended by the manufacturer: use the built-in spirit level, or a laboratory spirit level on the top of the piston to minimise the tilt. This should be checked also at full mass load.
- Use short, wide-bore pipework. This is more critical at low pressure.
- Ensure the cleanliness and the tightness of the tubing.
- Install an appropriate drainage system to control the nature of the fluid in the tubing.
- Attach a suitable temperature measurement system.

4.3 Pressure generation

4.3.1 For gas gauge pressure:

(a) Use a clean and dry gas (nitrogen for example), at a temperature near ambient.

(b) Adjust the pressure input to the range of the intercompared instruments.

(c) Clean the tubing of any liquid (for the oil-lubricated type).

4.3.2 For gas absolute pressure:

(a) Use a clean pump, or, when using mechanical rotational pumps, use an appropriate trap.

(b) Use an appropriate vacuum pump to ensure that the residual pressure over the mass-piston set is less than typically 10 Pa or 10^{-5} of the measured pressure, whichever is the higher, unless otherwise recommended by the manufacturer.

(c) Measure the residual pressure with a vacuum gauge calibrated and connected directly to the bell jar.

4.3.3 For liquid pressure:

(a) Use the liquid recommended by the manufacturer.

(b) If the liquid in the balance under calibration is not the same as the liquid in the standard, use an appropriate interface/separator to avoid any mixture of the two liquids.

(c) Clean the tubing of any other liquid.

(d) Clean the fluid in the tubing of any possible internal gas.

4.4 Pressure reference

4.4.1 The pressure reference instrument in general use for the calibration of a pressure balance is another pressure balance. For the ranges lower than 300 kPa, the standard instrument may be a mercury column manometer. Other instruments may be used as an alternative for specific cases (low gauge pressure, for example).

4.4.2 The calibration of an absolute pressure balance may be carried out in gauge mode, with an added uncertainty in A_0. However, the operation of the calibrated pressure balance in absolute mode should be tested.

4.4.3 The calibration of a pressure balance intended for the measurement of negative gauge pressures may be carried out in positive gauge pressure mode, with an added uncertainty in A_0. However, the operation of the calibrated pressure balance in negative gauge pressure mode should be tested.

4.4.4 In all cases, the reference instrument used for the calibration has to meet the following conditions:

(a) to be traceable to a national standard with a recognised calibration certificate.

(b) to have an uncertainty better than the presupposed uncertainty of the balance to be calibrated. Complete the uncertainty budget on the reference standard pressure balance to verify this condition.

4.5 Preparation of the pressure balance

4.5.1 The pressure balance under calibration shall be placed in the laboratory at least 12 hours before the calibration is started, to reach thermal equilibrium.

(a) Check that the oil is free of impurities. If not, drain all the tubing and replace the oil in the tank.

(b) With the pressure circuit closed and half the set of weights placed on the piston, the piston shall be moved upwards and downwards by means of the spindle pump. Thus, the mobility of the piston is examined over the total range of displacement.

(c) If necessary, and using the technical manual, remove the piston-cylinder assembly, and clean the surfaces of the two pieces with a suitable solvent or pure soap, and with a soft dry cloth according to the manufacturer's recommendations. Inspect the piston and the cylinder for surface scratches and corrosion. Relubricate the piston with clean liquid if the piston-cylinder operates in liquid, or if the balance operates in gas, but with an oil-lubricated piston-cylinder assembly.

(d) Examine the free rotation time (for the hand-rotating pressure balances only). Weights corresponding to 2/10 of maximum pressure are placed upon the piston. The initial rotation rate should be approximately 30 rpm. Measure the elapsed time until the piston is stationary. This time should be at least 3 min.

(e) Examine the descent rate of the piston. The piston descent rate is observed at maximum pressure when the piston is rotating. Measure the time interval in which the piston drops from top to bottom position. This time should be at least 3 min.

Note: For the last two parameters, the stated values should be related to the technical instructions of the manufacturer.

(f) Connect the pressure balance to the standard instrument.

(g) Identify the reference level for both pressure balances. The reference level is normally defined by the manufacturer at the bottom surface of the piston when it is in the working position. In the absence of reference level information, and when the bottom surface of the piston is not accessible, the reference level is generally defined at the outlet pipe connection level. The difference in height between the reference level of the standard and the reference level of the balance to be calibrated shall be reduced as much as possible and measured. In any case the difference in height between the reference levels of both the standard and the balance under calibration will need to be measured in order to apply the appropriate head correction (see paragraph 3).

(h) For absolute pressure, pump for 30 min. at the beginning of the calibration to eliminate the water vapour in the bell jar. Use dry nitrogen as the working gas.

(j) Rotate the piston or the cylinder while keeping to the manufacturer's recommendation.

(k) For hand-rotating balances, check the clockwise and anticlockwise direction influence (if any), or indicate the rotation direction in the certificate.

5 Example of calibration procedure

5.1 Methods to apply

Both methods that follow are comparative ones, consisting of comparing the balance to be calibrated and the standard instrument when both are submitted to the same pressure and the same environmental conditions. However, dependent on the presupposed accuracy of the balance to be calibrated, and according to customer requirements, alternative methods may be used:

5.1.1 Method A - Generated pressure method

The scope of this method is to determine the bias error and the repeatability of the calibrated pressure balance. This is done by determining the generated pressure corresponding to well-identified weights. In this method the weighing of the masses of the instrument under calibration is optional.

5.1.2 Method B - Effective area determination method

The scope of this method is to determine:

(a) the value of the mass of all the weights, including the piston of the pressure balance, if removable.

(b) the effective area A_p referred to 20 °C or another reference temperature t_r of the piston-cylinder assembly of the pressure balance as a function of pressure. At high pressure, this area can be expressed from the effective area at null pressure A_0 and the pressure distortion coefficient.

(c) the repeatability as a function of the measured pressure.

The elements relating to the determination of the effective area are given in section 6. The equations to be used for the calculation of the effective area are given in Appendix A.

Method A is usually not employed where the smallest uncertainty is required.

5.2 Method A procedure

5.2.1 At least three measuring series are carried out, each of them at pressures generated by the weights which the pressure balance under calibration is equipped with.

5.3 Method B procedure

5.3.1 Determination of the mass

(a) The value of the mass of each weight (including the floating elements if removable) of the pressure balance shall be determined by a laboratory accredited for such mass measurements. The relative uncertainty of the mass determination should not usually exceed 20 % of the likely total measurement uncertainty of the pressure balance to be calibrated. For example, if the supposed expanded uncertainty of the pressure balance is $5{\cdot}10^{-5} \times p$, the relative uncertainty of mass determination should be within $1{\cdot}10^{-5} \times m$.

(b) If the float base weight cannot be determined by weighing, the corresponding base pressure may be determined from the results of the pressure comparison measurements by using a least-squares analysis: in this case a tare value in pressure units should be given. The Δp-method mentioned in paragraph 6.3.3(c) allows the determination of this initial value.

5.3.2 Determination of the effective area

(a) For pressure balances which are equipped with both low pressure and high pressure piston-cylinder assemblies or with removable piston-cylinder assemblies, the complete calibration process should be carried out for each piston-cylinder assembly.

(b) The effective area shall be determined by carrying out at least three measuring series, each of them with at least six pressure points. The first point shall be chosen at the minimum value of the pressure range (manufacturer indicated value, or the lowest value corresponding to a satisfactory functioning, see paragraph 4.5, the latter being typically about 1/20 to 1/10 of the maximum pressure). The other pressure points should be spaced over the whole range between the minimum and the maximum pressure values.

Note: In the calibration certificate, the use range of the calibrated instrument can be stated along with the calibration range. If the use range begins below the calibration range, the uncertainty of the calibration stated in the calibration certificate must be given in such a form that, below the calibration range, the uncertainty never becomes smaller than at the lowest pressure of the calibration.

(c) The repeatability of the measured pressure is estimated from the experimental standard deviation calculated from the successive determinations operated for each pressure point.

Note (valid for both methods): Ascending measuring series can be considered to be identical to descending measuring series, as the balances used for pressure measurements usually have no significant hysteresis effect.

5.4 Cross-floating procedure

5.4.1 Gauge pressure mode

(a) When using a pressure balance as a standard instrument, the cross-floating method is carried out at each measuring point.

(b) Place the weights on the pressure balance to be calibrated, so that the masses correspond to the fixed pressure point.

(c) Adjust the pressure to equilibrate the balance under calibration.

(d) Perform an adjustment with small weights on one of both instruments (usually the one which is the more sensitive to a change in mass) if method B is used, or only on the reference pressure balance if method A is used, until the equilibrium condition of both balances has been found. The equilibrium should be considered as reached when the proper falling rate of both pistons is found (i.e. no flow of fluid in the tubing between the two pressure balances). Both pistons have to rotate during the adjustment. In the case of hand-rotating units, the influence of the clockwise/anticlockwise rotation, and of the spin rate will be checked.

(e) Note the reference number of each of the weights applied on both balances.

(f) Note the temperature of the piston-cylinder assembly of both balances. If the balance is not equipped with a temperature probe, note the surrounding air temperature using an electronic thermometer attached to some suitable point of the balance. This information shall be included in the certificate.

(g) Alternatively to the equilibrium control by monitoring the pistons' falling rate as described in (d), a differential pressure gauge can be used which is installed in the pressure line connecting two pressure balances under comparison (see 5.4.2). This method is particularly useful when a calibration is carried out in absolute pressure mode or when the compared pressure balances are operated with different fluids.

(h) Alternatively to the equilibrium controls described in (d) and (g), a high accuracy pressure gauge can be used. The measurement range of the gauge should cover the calibration pressure range. The pressure transducer measures the pressure generated by the reference and the pressure balance to be calibrated in turn. The difference of the successive reading gives the pressure difference between the two pressure balances (see 5.4.2). This method can be useful when calibrating in absolute pressure mode or when the compared pressure balances are operated with different fluids.

5.4.2 Absolute pressure mode

When using a pressure balance as a standard instrument, the cross-floating method can be used only if one of the pressure balances is equipped with a remote loading system including trim masses. If none of the cross-floated pressure balances has such a loading system, a differential pressure transducer equipped with a by-pass or an accurate absolute pressure gauge equipped with two valves can be used to measure the difference between the pressures measured by both balances.

When using a differential pressure transducer, for each pressure point:

(a) Open the bell jars, open the by-pass.

(b) Place the corresponding weights on both pressure balances.

(c) Adjust the pressure and masses to equilibrate the pressure balances.

(d) Read the zero of the transducer.

(e) Close and evacuate the bell jars. Close the by-pass.

(f) Adjust the pressure on both sides to equilibrate both balances.

(g) Record the reading of the transducer. If the differential pressure is so high that the needed uncertainty cannot be reached from the calibration of the transducer, repeat the last five operations.

(h) Note the reference number of each of the weights applied on both balances.

(i) Note the temperature of the piston-cylinder assembly of both balances. If the balance is not equipped with a temperature probe, note the surrounding air temperature.

(j) Note the residual pressure in the bell jar of both balances.

When using an accurate absolute pressure gauge connected to each pressure balance through a volume valve, for each pressure point:

(a) Open the bell jars, open both valves.

(b) Place the corresponding weights on both pressure balances.

(c) Adjust the pressure and masses to equilibrate the pressure balances.

(d) Close the valve of the balance under calibration.

(e) Close and evacuate the bell jars.

(f) Adjust the pressure on both sides to equilibrate both balances.

(g) Record the reading of the pressure gauge from the reference balance. Close the valve of the reference balance. Open the valve of the balance under calibration. Record the reading of the pressure gauge from the calibrated balance. If the difference of the two readings is so high that the needed uncertainty cannot be reached from the calibration of the pressure gauge, repeat the last five operations.

(h) Note the reference number of each of the weights applied on both balances.

(i) Note the temperature of the piston-cylinder assembly of both balances. If the balance is not equipped with a temperature probe, note the surrounding air temperature.

(j) Note the residual pressure in the bell jar of both balances.

6 Data evaluation and calibration certificate

6.1 General points

6.1.1 The calibration certificate shall be established in accordance with ISO 17025.

6.1.2 Preferably a separate calibration certificate shall be established for the determination of the mass of the weights. The identification of this mass certificate will be noted in the one related to the calibration of the pressure balance.

6.2 Method A procedure

6.2.1 The following technical data shall be included in the certificate:

(a) type of working fluid;

(b) linear thermal expansion coefficients of the piston-cylinder assembly under calibration (if not determined experimentally, e.g. using literature data, this shall be stated);

(c) position of the pressure reference level;

(d) information about how to convert the pressure values to the measurement temperature and to the local acceleration due to gravity.

6.2.2 Usually the results will be presented for the standard value of gravity 9.80665 $m \cdot s^{-2}$ (unless the customer requests his own local gravity) and the reference temperature (usually 20 °C) in the form of the table suggested in paragraph 6.2.2 as an example. It will include:

(a) the pressure indicated by the balance under calibration (p_m);

(b) the reference pressure measured by the standard instrument (mean of the repeated determinations), in Pa and in the unit of the pressure delivered by the balance, if different (p_r);

(c) the standard deviation of the reference pressure p_r;

(d) the difference between the indicated pressure and the reference pressure ($p_m - p_r$);

(e) the uncertainty of this difference, in the conditions of the calibration. The method used to estimate this uncertainty shall be reported in the certificate.

Indicated pressure	Mean reference pressure	Mean reference pressure	Experimental standard deviation of p_r	Pressure difference	Expanded uncertainty of pressure difference
p_m in X[(a)]	p_r in Pa[(b)]	p_r in X	$s(p_r)$ in X	p_m - p_r in X	$U(p_m - p_r)$ in X[(c)]

Notes

(a) X = Unit indicated by the pressure balance under calibration.

(b) This column may be replaced by a conversion factor in the certificate.

(c) The method of calculation of the uncertainty is described in section 7.

6.2.3 A table that lists all weights with their identification as applied on the calibrated unit at each pressure point of the calibration shall be included in the calibration certificate.

6.3 Method B procedure

6.3.1 The following technical data shall be included in the certificate:

(a) type of working fluid;

(b) equation according to which pressure can be calculated from the data reported in the certificate;

(c) linear thermal expansion coefficients of the piston-cylinder assembly under calibration (if not determined experimentally, e.g. by using literature data, this shall be stated);

(d) position of the pressure reference level;

(e) volume for fluid buoyancy correction when this is required.

6.3.2 The results of the calibration, after analysis (see below):

(a) effective area and its combined uncertainty;

(b) if relevant, the pressure distortion coefficient(s) and the corresponding combined uncertainty.

6.3.3 Calculation of the effective area:

(a) The computing method described in detail in Appendix A can be used to calculate the effective area of the pressure balance to be calibrated from the mass applied on its piston and the pressure delivered by the standard instrument.

(b) In this method, the effective area is calculated by reversing the equation of pressure presented in section 3.

(c) The use of other methods, such as the differential method (Δp-method) to eliminate potential zero-errors is not excluded, but requires some experience in the analysis of the results. Particularly the Δp-method may be one of the possible methods if method B is used for the determination of the effective area of pressure balances with an unknown initial weight that cannot be determined by weighing.

(d) The effective area values determined for every pressure point allow a modelling of the effective area as a function of pressure.

(e) The results may be presented in the form of the table below, suggested as a comprehensive example, and including:

(i) the reference pressure measured by the reference standard instrument in each pressure point, in Pa and in the unit of the pressure delivered by the balance, if different;

(ii) the corresponding mass applied on the floating element of the balance to be calibrated;

(iii) the corresponding temperature of the measuring assembly during the calibration;

(iv) the individual value of the effective area A_p calculated at the reference temperature and at reference pressure, as described in Appendix A;

(v) the mean value of the effective area A_p;

(vi) the expanded uncertainty of A_p.

Reference pressure p_r kPa	Mass applied on the piston kg	Temperature of the assembly °C	Effective area at t_r and p_r mm²	Mean effective area (n = 5) mm²	Expanded uncertainty of the mean effective area mm²
400.096	6.162 52	21.28	156.931		
400.083	6.162 52	20.86	156.937		
400.083	6.162 52	20.88	156.938	156.940	0.015
400.063	6.162 52	20.86	156.948		
400.078	6.162 52	20.80	156.944		
		.....			

(f) Then the effective area as a function of pressure is analysed using the least-squares method. Three cases may be observed:

(i) the dependence upon pressure is not significant relating to the standard deviation (this is usually the case for the low-range pressure balances). The effective area at null pressure A_0 is calculated as the mean value of all the determinations. If the theoretical pressure distortion coefficient is known, it shall be used for calculating the effective area. The type A standard uncertainty is estimated from the experimental standard deviation of the distribution.

(ii) the dependence upon pressure can be considered to be linear. The effective area at null pressure A_0 and the pressure distortion coefficient λ are calculated as parameters of the least-squares straight line. The type A combined standard uncertainty of A_p is estimated using the variances and the covariance of A_0 and λ.

(iii) the dependence upon pressure cannot be considered to be linear. The effective area at null pressure A_0 and the pressure distortion coefficient λ (first order) and λ' (second order) are calculated by the least-squares second-order fit. The type A combined standard uncertainty is estimated using the variances and the covariances of A_0, λ and λ'.

(g) The variances and the covariances of the parameters shall be estimated using literature on statistics. For the linear model, equations are presented in Appendix A.

(h) The certificate shall report:

(i) the calculated value of the effective area under reference conditions A_0 and the corresponding uncertainty, estimated from the standard deviation of A_0, the contribution of the standard, the measurement of the mass applied to the moving element, the temperature and other measured quantities.

(ii) when relevant, the pressure distortion coefficient(s) and the uncertainty of A_p estimated from the variances and the covariance(s) of A_0 and λ(s), the contribution of the standard, the measurement of the mass applied to the moving element, the temperature and other measured quantities.

6.3.4 Calculation of the measured pressure:

(a) The pressure measured by the pressure balance to be calibrated can be calculated using the equations presented in section 3. It is useful to the user to have this measured pressure compared to the reference pressure delivered by the standard, under the conditions of the calibration.

(b) The results shall be presented in the form of the table below suggested as an example and including:

(i) the reference pressure measured by the standard instrument, in Pa and in the unit of the pressure delivered by the balance if different;

(ii) the corresponding pressure measured by the balance under calibration, and calculated from the data (effective area and pressure distortion coefficient) taken from the certificate;

(iii) the difference between the measured pressure and the reference pressure for each pressure equilibrium, as residuals of the effective area modelling;

(iv) the mean value of these differences;

(v) the experimental standard deviation of the measured differences.

(c) This table gives information on a potential residual pressure due to unknown forces and on the repeatability of the pressure balance as a function of pressure. Hence, the minimum information contained in this part of the certificate is the mean difference and the experimental standard deviation.

Reference pressure p_r **KPa**	**Measured pressure p_m** **kPa**	**Difference p_m - p_r** **kPa**	**Mean difference (n = 5)** **kPa**	**Experimental standard deviation of p_m - p_r** **kPa**
600.152	600.159	+ 0.000 6		
600.155	600.161	+ 0.000 6		
600.149	600.161	+ 0.001 1	+ 0.001 8	0.001 7
600.114	600.161	+ 0.004 6		
600.140	600.161	+ 0.002 1		
		....		

7 Estimation of the uncertainty

The combined uncertainty of the measured pressure (calibration method A) or of the effective area (calibration method B) under the conditions of the calibration shall be estimated in conformity with the document JCGM 100:2008 (GUM). The components to be taken into account are listed below for both recommended methods.

7.1 Method A

7.1.1 Estimation of type A uncertainty (u_A components):

(a) Repeatability of the balance, estimated as a function of pressure from the values of the standard deviation of the pressure difference expressed in the table in 6.2.2. Following the experimental data, it can be expressed in Pa, or using a term proportional to the pressure, or both terms.

7.1.2 Estimation of type B uncertainty (u_B components):

(a) Uncertainty of the reference pressure;

(b) Uncertainty due to temperature;

(c) Uncertainty due to the head correction;

(d) Uncertainty due to tilt (negligible if perpendicularity was duly checked);

(e) Uncertainty due to spin rate and/or direction, if applicable;

(f) Uncertainty of the residual pressure (absolute mode only);

(g) Uncertainty due to limited cross-floating sensitivity (required when there is no reliable estimation of the repeatability of the pressure balance).

7.1.3 When the standard uncertainty is estimated for each component, the combined standard uncertainty, and then the expanded uncertainty are calculated in conformity with the publication JCGM 100:2008 (GUM).

7.2 Method B

7.2.1 Estimation of type A uncertainty (u_A components):

(a) Repeatability of the balance, estimated as a function of pressure from the values of the standard deviation of the effective area as expressed in the table in 6.3.3. Alternatively, the type A uncertainty of pressure can be presented by an equation based on the variances and covariance of A_0 and λ.

7.2.2 Estimation of type B uncertainty (u_B components):

(a) Uncertainty of the reference pressure;

(b) Uncertainty of the masses;

(c) Uncertainty due to the temperature of the balance;

(d) Uncertainty due to the thermal expansion coefficient of the piston-cylinder assembly;

(e) Uncertainty due to the air buoyancy;

(f) Uncertainty due to the head correction;

(g) Uncertainty due to the surface tension of the pressure-transmitting fluid;

(h) Uncertainty due to tilt (negligible if perpendicularity was duly checked);

(i) Uncertainty due to spin rate and/or direction, if applicable;

(j) Uncertainty of the residual pressure (absolute mode only).

7.2.3 When the standard uncertainty is estimated for each component, the combined standard uncertainty, and then the expanded uncertainty, are calculated in compliance with the document JCGM 100:2008 (GUM). An example of an uncertainty budget corresponding to the use of a pressure balance calibrated using method B is given in Appendix B.

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Appendix A

Computing method used to determine the effective area with the associated uncertainty of the piston-cylinder assembly of a pressure balance

A1 The determination of the effective area of the piston-cylinder assembly of a pressure balance is based on the equation for pressure generated by the pressure balance at its reference level, which is established by analysing the forces applied to the piston. The following expression is given as an example. It corresponds to the case of a gas-operated balance, in gauge mode. The calculation progress would be the same for other types of pressure balances (see section 4).

$$p = \frac{\sum_i m_i \cdot g \cdot (1 - \rho_{\mathrm{a}} / \rho_{m_i})}{A_{\mathrm{p}} \cdot [1 + (\alpha_{\mathrm{p}} + \alpha_{\mathrm{c}}) \cdot (t - t_{\mathrm{r}})]}, \tag{A.1}$$

where:

- p is the gauge pressure measured at the bottom of the piston,
- m_i are the individual mass values of the weights applied on the piston, including all floating elements,
- g is the local gravity,
- ρ_{a} is the density of air,
- ρ_{mi} are the densities of the weights. If the weights are made of different materials, it is necessary to take the different densities into account,
- A_{p} is the effective area of the piston-cylinder assembly at the reference temperature t_{r} as a function of pressure,
- α_{p} is the linear thermal expansion coefficient of the piston,
- α_{c} is the linear thermal expansion coefficient of the cylinder,
- t is the temperature of the piston-cylinder assembly,
- t_{r} is the reference temperature of the piston-cylinder assembly.

A2 Using the cross-floating method, two inter-compared balances, in equilibrium conditions, measure the same pressure. Therefore, for each calibration point, referenced by suffix j, corresponding to a total mass Σm_i, the reference pressure p_j generated by the reference pressure balance at the reference level of the calibrated pressure balance is calculated from equation (A.1) above, by using the known characteristics of the reference instrument, and by adding the head correction between the reference levels of both instruments. Then, from this pressure p_j, the effective area $A_{\mathrm{p}j}$ at the reference temperature t_{r} (usually 20 °C) of the balance to be calibrated is determined for each pressure p_j, by:

$$A_{\mathrm{p}j} = \frac{\sum_i m_{ij} \cdot g \cdot (1 - \rho_{\mathrm{a}j} / \rho_{m_{ij}})}{p_j \cdot [1 + (\alpha_{\mathrm{p}} + \alpha_{\mathrm{c}}) \cdot (t_j - t_{\mathrm{r}})]}, \tag{A.2}$$

in which $\Sigma_i m_{ij}$ is the overall mass, t_j the temperature, and α_{p}, α_{c}, ρ_{mij} the characteristics of the balance under calibration, $\rho_{\mathrm{a}j}$ is the air density, with all quantities bearing index j, being specific for pressure point p_j.

From the analysis of the mean results $A_p(p)$, where $A_p(p)$ is the average of all effective areas A_{pj} determined at the same nominal reference pressure p_r, $A_p(p) = \langle A_{pj}(p) \rangle$, three cases can arise:

1 The effective area $A_p(p)$ is independent of pressure. This can be considered the case when the variation of $A_p(p)$ over the calibration pressure range is comparable with the standard deviations of $A_{pj}(p)$ calculated at each nominal reference pressure. In this case, the effective area under reference conditions is equal to the mean of all N determinations. The type A standard uncertainty of the effective area, $u_A(A_p)$, is the standard deviation of the distribution of all these determinations.

$$A_p = A_0 = \sum A_{pj} \Big/ N \tag{A.3}$$

$$u_A(A_p) = \left[\sum \left(A_{pj} - A_0\right)^2 \Big/ (N-1)\right]^{0.5} \tag{A.4}$$

2 The effective area $A_p(p)$ is a linear function of pressure. This can be considered the case when the deviations of $A_p(p)$ from the best fit straight line of $(A_{pj};p_j)$ are comparable with the standard deviations of $A_{pj}(p)$ calculated at each nominal reference pressure. Then noting A_0, the effective area at null pressure, and λ, the pressure distortion coefficient of the piston-cylinder assembly, $A_p(p)$ can be presented by equation:

$$A_p = A_0 \cdot (1 + \lambda \cdot p) \,, \tag{A.5}$$

where A_0 and λ are defined by expressions:

$$A_0 = \frac{\sum p_j^2 \cdot \sum A_{pj} - \sum p_j \cdot \sum p_j \cdot A_{pj}}{N \cdot \sum p_j^2 - \left(\sum p_j\right)^2} \quad \text{and} \quad \lambda = \theta_1 / A_0 \tag{A.6}$$

with θ_1 being the slope of the linear fit:

$$\theta_1 = \frac{N \cdot \sum p_j \cdot A_{pj} - \sum p_j \cdot \sum A_{pj}}{N \cdot \sum p_j^2 - \left(\sum p_j\right)^2} \,. \tag{A.7}$$

The type A standard uncertainties of A_0, λ and A_p corresponding to the distribution of A_{pj} around the fit function $A_p(p)$ are calculated from the variances and the covariance of A_0 and θ_1, $V(A_0)$, $V(\theta_1)$ and $\text{cov}(A_0, \theta_1)$:

$$u_A(A_0) = \left[N \cdot V(A_0)\right]^{0.5} \tag{A.8}$$

$$u_A(\lambda) = \left[N \cdot V(\theta_1)\right]^{0.5} \Big/ A_0 \tag{A.9}$$

$$u_A(A_p) = N^{0.5} \cdot \left[V(A_0) + V(\theta_1) \cdot p^2 + 2 \cdot \text{cov}(A_0, \theta_1) \cdot p\right]^{0.5} \tag{A.10}$$

with

$$V(A_0) = \frac{\sum p_j^2}{N \cdot \sum p_j^2 - \left(\sum p_j\right)^2} \cdot \frac{\sum \left(A_{pj} - A_0 - \theta_1 \cdot p_j\right)^2}{N-2} \tag{A.11}$$

$$V(\theta_1) = \frac{N}{N \cdot \sum p_j^2 - \left(\sum p_j\right)^2} \cdot \frac{\sum \left(A_{pj} - A_0 - \theta_1 \cdot p_j\right)^2}{N-2} \tag{A.12}$$

$$\text{cov}(A_0, \theta_1) = \frac{-\sum p_j}{N \cdot \sum p_j^2 - \left(\sum p_j\right)^2} \cdot \frac{\sum \left(A_{pj} - A_0 - \theta_1 \cdot p_j\right)^2}{N-2} \,. \tag{A.13}$$

Note: Type A uncertainties of A_0, λ and A_p as presented by equations (A.4, A.8, A.9 and A.10) correspond to the standard deviation of the distribution of the measurement points around the mean value or the fit straight line and, thus, are the standard deviations of a single measurement. The use of the standard deviation of the single value instead of the standard deviation of the mean is motivated by the fact that the measurement data are not independent, strongly correlate in pressure and frequently show systematic deviations from the assumed models. The uncertainties as defined by equations (A.4, A.8, A.9 and A.10) are invariant to the number of measurement points and reflect the uncertainty expected when the pressure balance is used for a single pressure measurement.

3 The effective area $A_p(p_r)$ is a non-linear function of pressure. This takes place when the deviations of $A_p(p_r)$ from the best fit straight line of $(A_{p\,j};p_{r\,j})$ are significant compared with the standard deviations of $A_{p\,j}(p_r)$ calculated at each nominal reference pressure. Then the effective area can be presented by a second-order polynomial expression:

$$A_p = A_0 \cdot (1+\lambda \cdot p+\lambda' \cdot p^2)\,. \tag{A.14}$$

A_o, λ and λ' are also calculated using the least-squares method.

Note: Particular attention should be paid to the experimental data before this model is applied, because it implies that the pressure distortion coefficient depends on pressure. In fact there can be various reasons for a non-linearity of the experimental data, whose analysis can require models different from that presented by equation (A.14). In particular, a non-linearity of $(A_{pj};p_j)$ increasing at lower pressures may deal with a constant force error caused e.g. by erroneous head correction, mass of the piston and weight carrier, surface tension effect, etc., and can only be badly analysed with the model (A.14).

The methods utilising equations (A.3, A.4, A.6, A.7, A.11 – A.13) imply that all measurement data are characterised by the same uncertainty of all $A_{p\,j}$. Generally the uncertainty of $A_{p\,j}$ can significantly change with pressure. In practice, a non-linearity or a larger scatter in the experimental data may appear at low pressure (s. *Note* in A2, 3). In order to take into account the varying reliability of the experimental data and to optimise the calculation of the effective area and its uncertainty, the weighted least squares method can be used. Weights (g_j) associated with each data point $A_{p\,j}$ and derived from its uncertainties are incorporated into the fitting process. As uncertainty sources, those can be considered which cause deviations of $A_{p\,j}$ from the model equations: mass, head correction, temperature, sensitivity of the pressure balance, oil surface tension, residual pressure, etc. as well as random variations of $A_{p\,j}$ values obtained at the same nominal pressure characterised by their standard deviations. The value of each weight g_j is determined as the reciprocal sum of their squared uncertainty contributions $u_i^2(A_{p\,j})$:

$$g_j = \frac{1}{\sum_i u_i^2\left(A_{pj}\right)}, \tag{A.15}$$

and the effective area with its uncertainty ascribed to the fit can be calculated as follows.

For case 1 of effective area $A_p(p)$ being independent of pressure, the following equations (A.3a, A.4a) are taken instead of (A.3, A.4):

$$A_p = A_0 = \sum_j g_j \cdot A_{pj} \Big/ \sum_j g_j \tag{A.3a}$$

$$u_A\left(A_p\right) = \left(N \Big/ \sum_j g_j\right)^{0.5}. \tag{A.4a}$$

For case 2 of effective area $A_p(p_r)$ being a linear function of pressure, equations (A.6, A.7, A.11-A.13) are replaced by equations (A.6a, A.7a, A.11a-A.13a), respectively:

$$A_0 = \frac{\sum g_j \cdot p_j^2 \cdot \sum g_j \cdot A_{pj} - \sum g_j \cdot p_j \cdot \sum g_j \cdot p_j \cdot A_{pj}}{\sum g_j \cdot \sum g_j \cdot p_j^2 - \left(\sum g_j \cdot p_j\right)^2} \tag{A.6a}$$

$$\theta_1 = \frac{\sum g_j \cdot \sum g_j \cdot p_j \cdot A_{pj} - \sum g_j \cdot p_j \cdot \sum g_j \cdot A_{pj}}{\sum g_j \cdot \sum g_j \cdot p_j^2 - \left(\sum g_j \cdot p_j\right)^2} \tag{A.7a}$$

$$V(A_0) = \frac{\sum g_j \cdot p_j^2}{\sum g_j \cdot \sum g_j \cdot p_j^2 - \left(\sum g_j \cdot p_j\right)^2} \tag{A.11a}$$

$$V(\theta_1) = \frac{\sum g_j}{\sum g_j \cdot \sum g_j \cdot p_j^2 - \left(\sum g_j \cdot p_j\right)^2} \tag{A.12a}$$

$$\mathrm{cov}(A_0, \theta_1) = \frac{-\sum g_j \cdot p_j}{\sum g_j \cdot \sum g_j \cdot p_j^2 - \left(\sum g_j \cdot p_j\right)^2}. \tag{A.13a}$$

Type A standard uncertainties of A_0, λ and A_p corresponding to the distribution of A_{pj} around the fit function $A_p(p)$ are calculated by equations (A.8, A.9 and A.10).

Note: In contrast to the non-weighted least squares method, the uncertainty formulae for the weighted least squares method (A.11a, A.12a and A.13a) are based only on the assumed uncertainties u_i,, do not contain differences between the experimental data and the model functions and, thus, cannot serve as a goodness-of-fit measure. A chi-squared test should be applied to carry out a consistency check for the measurement data with their uncertainties and the model used. Only if the consistency check succeeds, can the results of the fit with their uncertainties be considered as representative for the measurement results.

A3 The type B uncertainty estimation for effective area A_p is performed by analysing it as a function of input quantities X_i used for the A_p calculation, $A_p = A_p(X_1, X_2, ..., X_n)$. Corresponding to the type of pressure balance and the operation mode, equations (3.1), (3.2) or (3.3) are used with X_i being all the quantities which appeared in these equations. In addition, a correction factor for a possible tilt angle Θ of the piston from the verticality is added to the gravitational force for the purpose of the uncertainty estimation. For the calculation of the effective area, Θ is presumed to be zero. The procedure for the uncertainty calculation is as follows:

(a) Estimate the uncertainty $U(X_i)$ of each component. For some influence quantities their uncertainties can be known, for some of them they can be estimated from the bounds of the quantities' variation.

(b) Determine the standard uncertainty $u(X_i)$ from the probability distribution of quantity X_i.

(c) Determine the standard uncertainty $u_i(A_p)$ due to the quantity X_i using the sensitivity coefficient calculated as the partial derivative of the function $A_p(X_1, X_2, ..., X_n)$ with respect to quantity X_i:

$$u_i(A_p) = \left| \frac{\partial A_p(X_1, X_2, ..., X_n)}{\partial X_i} \right| \cdot u(X_i). \tag{A.16}$$

(d) Calculate the type B standard uncertainty $u_B(A_p)$ according to:

$$u_B(A_p) = \left[\sum_i^n u_i^2(A_p)\right]^{0.5} . \tag{A.17}$$

(e) Calculate the combined standard uncertainty $u(A_p)$ as

$$u(A_p) = \left[u_A^2(A_p) + u_B^2(A_p)\right]^{0.5} , \tag{A.18}$$

where $u_A(A_p)$ is the type A uncertainty defined, in dependence on the model and the method used, by equation (A.4), (A.10) or (A.4a).

(f) Calculate the expanded uncertainty $U(A_p)$ by multiplying the standard uncertainty by the coverage factor $k = 2$

$$U(A_p) = 2 \cdot u(A_p) . \tag{A.19}$$

Note: If the weighted mean (equation A.4a) or the weighted least squares method (equation A.10 with the parts defined by equations A.11a, A.12a and A.13a) were used for the calculation of the type A uncertainty, only those components X_i should be included into the type B uncertainty calculation which have not been used for the calculation of weights g_j (equation A.15).

An example of the uncertainty calculation for the effective area is presented in Appendix B.

Appendix B

Example of uncertainty estimation for the effective area of a pressure balance

B1 **Scope**

This example presents the calculation of the effective area and the associated uncertainty of an oil-operated pressure balance having a significant pressure distortion coefficient when it is calibrated using another pressure balance as a reference. This example corresponds to calibration method B. The evaluation of the effective area and the pressure distortion coefficient with their type A uncertainties is performed using the ordinary (non-weighted) least squares method. The estimation of the type B uncertainty is based on the measurement procedure, the uncertainty of the reference pressure generated with the reference pressure balance, the data included in the calibration certificates of the balance under calibration and the environmental conditions.

B2 **Definition of the effective area**

The general definition of the effective area A_p, at the reference temperature t_r, of an oil-operated pressure balance to be calibrated in gauge mode by a reference pressure standard, which is presumed to be a pressure balance too, is given by the following expression:

$$A_\mathrm{p} = \frac{\left[\sum_i m_i \cdot g \cdot (1 - \rho_\mathrm{a} / \rho_{m_i}) + \sigma \cdot c\right] \cdot \cos(\Theta)}{\left[p_\mathrm{r} + \left(\rho_\mathrm{f} - \rho_\mathrm{a}\right) \cdot g \cdot \Delta h\right] \cdot \left[1 + (\alpha_\mathrm{p} + \alpha_\mathrm{c}) \cdot (t - t_\mathrm{r})\right]}, \qquad \text{(B.1)}$$

where:

p_r is the pressure generated by the reference pressure standard at its reference level;

m_i are the individual mass values of the weights applied on the piston, including all floating elements;

g is the local gravity;

ρ_a is the density of air;

ρ_{mi} are the densities of the weights. If the weights are made of different materials, it is necessary to take the different densities into account;

α_p is the linear thermal expansion coefficient of the piston;

α_c is the linear thermal expansion coefficient of the cylinder;

t is the temperature of the piston-cylinder assembly;

σ is the surface tension of the oil;

c is the circumference of the piston;

ρ_f is the density of the measuring fluid;

Δh is the difference between the altitude h_1 of the reference level of the reference pressure standard and the altitude h_2 of the reference level of the pressure balance under calibration: $\Delta h = h_1 - h_2$, with $h_1 > h_2$ if the level of the reference standard is higher than that of the pressure balance under calibration. In some cases, the reference level of the pressure balance is a function of the oil buoyancy of the piston: the exact reference level is to be indicated in the calibration certificate.

Θ is the angle of deviation of the piston axis from verticality. For a properly levelled pressure balance this angle is equal to zero, however, its uncertainty should be considered in the uncertainty budget.

The results to be reported in the calibration certificate are the values of the zero-pressure effective area (A_0), the pressure distortion coefficient of the piston-cylinder assembly (λ), and the individual value of the mass of each weight. The calibration certificates also give the corresponding expanded uncertainties of each parameter, including those of the pressure-dependent effective area A_p based on A_0 and λ.

B3 Calculation of the effective areas, the pressure distortion coefficient and their type A standard uncertainties

In the table below, the effective areas A_{pj} are presented as calculated with equation (B.1) from data obtained in five measurement series in the pressure range from 50 MPa to 500 MPa.

Nominal pressure (p_j) in MPa	Effective area (A_{pj}) in mm^2				
	Series 1	Series 2	Series 3	Series 4	Series 5
50	1.961069	1.961057	1.961080	1.961076	1.961082
100	1.961201	1.961196	1.961208	1.961196	1.961201
150	1.961325	1.961316	1.961324	1.961316	1.961321
200	1.961431	1.961425	1.961425	1.961420	1.961424
250	1.961530	1.961525	1.961529	1.961527	1.961529
300	1.961627	1.961626	1.961629	1.961623	1.961621
350	1.961722	1.961723	1.961719	1.961716	1.961715
400	1.961816	1.961816	1.961814	1.961810	1.961805
450	1.961909	1.961909	1.961904	1.961901	1.961894
500	1.962008	1.962008	1.961999	1.962001	1.961982

It is assumed that the head corrected pressures p, $p = p_r + (\rho_f - \rho_a)\cdot g\cdot\Delta h$, do not differ much in the five series and that they are very close to the nominal values. The model of the effective area linearly depending on pressure according to equation (A.5) is used. Substituting p_j and A_{pj} values in equations (A.6 to A.13), $N = 50$, the following results are obtained

with equations (A.6, A.7):

$$A_0 = 1.961004 \text{ mm}^2, \qquad \theta_1 = 2.024\cdot10^{-6} \text{ mm}^2\cdot\text{MPa}^{-1} \qquad \lambda = 1.032\cdot10^{-6} \text{ MPa}^{-1},$$

with equations (A.11, A.12, A.13):

$$V(A_0) = 2.7\cdot10^{-11} \text{ mm}^4, \quad V(\theta_1) = 2.8\cdot10^{-16} \text{ mm}^4\cdot\text{MPa}^{-2}, \quad \text{cov}(A_0,\theta_1) = -7.7\cdot10^{-14} \text{ mm}^4\cdot\text{MPa}^{-1},$$

with equations (A.8, A.9):

$$u_A(A_0)/A_0 = 1.9\cdot10^{-5}, \qquad u_A(\lambda) = 6.0\cdot10^{-8} \text{ MPa}^{-1},$$

and with equation (A.10):

$$u_A(A_p)/A_p = [(1.9\cdot10^{-5})^2 + (6.0\cdot10^{-8})^2\cdot(p/\text{MPa})^2 - 2\cdot(1.0\cdot10^{-6})^2\cdot(p/\text{MPa})]^{0.5}.$$

B4 Calculation of type B standard uncertainty of the effective area

Type B standard uncertainty is calculated following the procedure given in Appendix A3. The uncertainty sources and their contributions to the effective area uncertainty are considered below.

B4.1 – *Reference pressure*

The uncertainty of the pressure generated by the reference pressure balance is calculated on the basis of data given in the calibration certificate of this pressure balance taking into account the measurement conditions of the laboratory. For a pressure balance operated in the range of 500 MPa, the uncertainty of its pressure $U(p_r)$ can be presented as a geometric sum of three terms – constant, pressure-proportional and squared-pressure-proportional.

For example: $U(p_r) = [(10^{-4})^2 + (4{\cdot}10^{-5})^2{\cdot}(p/\text{MPa})^2 + (2{\cdot}10^{-7})^2{\cdot}(p/\text{MPa})^4]^{0.5}$ MPa.

If $U(p_r)$ is expressed using a coverage factor $k = 2$:

$$\frac{u_1(A_p)}{A_p} = \frac{u_{p_r}(A_p)}{A_p} = \frac{1}{p_r + (\rho_f - \rho_a)\cdot g\cdot \Delta h}\cdot\frac{U(p_r)}{2} \approx \frac{U(p_r)}{2\cdot p}.$$

With the example of $U(p_0)$ given above:

$$u_1(A_p)/A_p = [(5{\cdot}10^{-5})^2{\cdot}(p/\text{MPa})^{-2} + (2{\cdot}10^{-5})^2 + (1{\cdot}10^{-7})^2{\cdot}(p/\text{MPa})^2]^{0.5}.$$

B4.2 – *Mass*

The mass values of the weights given in the calibration certificate are used to calculate the total mass applied to the piston. The uncertainty of the total mass is taken as the arithmetic sum of the uncertainties of the weights' masses $U(m_i)$, because they can be considered as correlated. If $U(m_i)$ are expressed at $k = 2$:

$$\frac{u_2(A_p)}{A_p} = \frac{u_{\sum_i m_i}(A_p)}{A_p} = \frac{g\cdot(1-\rho_a/\rho_{m_i})}{\sum_i m_i\cdot g\cdot(1-\rho_a/\rho_{m_i}) + \sigma\cdot c}\cdot\frac{\sum_i U(m_i)}{2} \approx \frac{\sum_i U(m_i)}{2\cdot\sum_i m_i}.$$

For example: $u_2(A_p)/A_p = 2.6{\cdot}10^{-6}$.

B4.3 – *Temperature of the piston-cylinder assembly*

The temperature of the piston-cylinder assembly is measured using a temperature probe. The uncertainty of the measurement, including the calibration uncertainty of the measuring instrument, and the temperature gradient in the pressure balance is ± 0.5 °C. Assuming a rectangular probability distribution for the temperature:

$$\frac{u_3(A_p)}{A_p} = \frac{u_t(A_p)}{A_p} = \frac{(\alpha_p + \alpha_c)}{1 + (\alpha_p + \alpha_c)\cdot(t - t_r)}\cdot\frac{U(t)}{\sqrt{3}} \approx (\alpha_p + \alpha_c)\cdot\frac{U(t)}{\sqrt{3}}.$$

For example, if both piston and cylinder are made of tungsten carbide and $\alpha_p + \alpha_c = 9{\cdot}10^{-6}$ °C^{-1}:

$$u_3(A_p)/A_p = 2.6{\cdot}10^{-6}.$$

B4.4 – *Thermal expansion coefficient of the piston-cylinder assembly*

The uncertainty of the thermal expansion coefficient of the piston and the cylinder contributes to the uncertainty of the effective area when the temperature of the piston-cylinder assembly deviates from the reference temperature. With the difference between the two temperatures $|t - t_r|$ and the relative expanded uncertainty of $\alpha_p + \alpha_c$, $U_{rel}(\alpha_p + \alpha_c)$, known at $k = 2$:

$$\frac{u_4(A_p)}{A_p} = \frac{u_{\alpha_p+\alpha_c}(A_p)}{A_p} = \frac{|t - t_r|\cdot(\alpha_p + \alpha_c)}{1 + (\alpha_p + \alpha_c)\cdot(t - t_r)}\cdot\frac{U_{rel}(\alpha_p + \alpha_c)}{2} \approx |t - t_r|\cdot(\alpha_p + \alpha_c)\cdot\frac{U_{rel}(\alpha_p + \alpha_c)}{2}.$$

For example, if $|t - t_r| = 2$ °C, $\alpha_p + \alpha_c = 9{\cdot}10^{-6}$ °C^{-1} and $U_{rel}(\alpha_p + \alpha_c) = 10$ %:

$$u_4(A_p)/A_p = 9{\cdot}10^{-7}.$$

B4.5 – *Local gravity acceleration*

As the gravity acceleration applies to both the standard and the pressure balance under calibration, its uncertainty should not contribute to the uncertainty of the effective area. As the uncertainty of the reference pressure, whose contribution was considered in B4.1 already includes the gravity acceleration as one of the uncertainty sources (see C4.5), its uncertainty should be subtracted from the combined uncertainty of the effective area. If the uncertainty of the gravity acceleration $U(g)$ is expressed at k = 3:

$$\frac{u_5(A_p)}{A_p} = \frac{u_g(A_p)}{A_p} \approx \frac{\sum_i m_i \cdot (1-\rho_a/\rho_{m_i})}{\sum_i m_i \cdot g \cdot (1-\rho_a/\rho_{m_i}) + \sigma \cdot c} \cdot \frac{U(g)}{3} \approx \frac{U(g)}{3 \cdot g}.$$

For example, if the gravity acceleration has been determined by calculation from the local longitude, latitude and altitude, its uncertainty expressed at k = 3 is $U(g) = 4{\cdot}10^{-5} \times g$:

$$u_5(A_p)/A_p = 1.3{\cdot}10^{-5}.$$

Note: This uncertainty should be geometrically subtracted from the combined standard uncertainty of the effective area.

B4.6 – *Air buoyancy – density of air*

It is supposed in this example that the value of the air density ρ_a is calculated from the measured values of the atmospheric pressure, and the ambient temperature and humidity, using a simplified formula providing the uncertainty of ρ_a, $U(\rho_a) = 5{\cdot}10^{-3} \times \rho_a$, expressed at k = 2:

$$\frac{u_6(A_p)}{A_p} = \frac{u_{\rho_a}(A_p)}{A_p} \approx \frac{\sum_i m_i \cdot g/\rho_{m_i}}{\sum_i m_i \cdot g \cdot (1-\rho_a/\rho_{m_i}) + \sigma \cdot c} \cdot \frac{U(\rho_a)}{2} \approx \frac{\sum_i m_i/\rho_{m_i}}{\sum_i m_i} \cdot \frac{U(\rho_a)}{2}.$$

For example, with the nominal air density of ρ_a = 1.2 kg·m^{-3} and the weights' density being the same for all main weights and equal to ρ_{mi} = 7920 kg·m^{-3}, the uncertainty contribution will be:

$$u_6(A_p)/A_p = 3.8{\cdot}10^{-7}.$$

B4.7 – *Air buoyancy – density of weights*

Usually the mass of the weights is determined by weighing them in the air so that the uncertainty of the mass already includes the uncertainty of the weight density. Moreover, the error in the mass of a weight coming from its uncertain density is compensated to a high extent when this weight is used for loading the piston. The uncertain density would have no effect on the pressure if the air density during the mass determination and pressure measurement were the same. At different air densities during the weighing and the pressure measurement, the effect of the uncertain weight density will depend on the difference of the air densities $\Delta\rho_a$. At extremely different conditions, e.g. if the mass determination and pressure measurement locations differ by up to 3000 m in height, the air density difference can be as high as $\Delta\rho_a$ = 0.4 kg·m^{-3}. If the density uncertainty of all the main weights is the same and equal to $U(\rho_{mi})$ = 40 kg·m^{-3}, expressed at k = 2, the uncertainty contribution will be:

$$\frac{u_7(A_p)}{A_p} = \frac{u_{\rho_{m_i}}(A_p)}{A_p} = \frac{\Delta\rho_a \cdot \sum_i m_i \cdot g/\rho_{m_i}^2}{\sum_i m_i \cdot g \cdot (1-\rho_a/\rho_{m_i}) + \sigma \cdot c} \cdot \frac{U(\rho_{m_i})}{2} \approx \frac{\Delta\rho_a}{\rho_{m_i}^2} \cdot \frac{U(\rho_{m_i})}{2}.$$

For example, with the density being the same for all the main weights and equal to ρ_{mi} = 7920 kg·m^{-3}, the uncertainty contribution will be:

$$u_7(A_p)/A_p = 1.3{\cdot}10^{-7}.$$

B4.8 – *Head correction – height difference*

The head correction is calculated from three parameters ρ_f, g and Δh. If the uncertainty of the Δh measurement is $U(\Delta h)$ = 2 mm, expressed at k = 2, the uncertainty contribution will be:

$$\frac{u_8(A_p)}{A_p} = \frac{u_{\Delta h}(A_p)}{A_p} = \frac{(\rho_f - \rho_a)\cdot g}{p_r + (\rho_f - \rho_a)\cdot g \cdot \Delta h} \cdot \frac{U(\Delta h)}{2} \approx \frac{\rho_f \cdot g}{p} \cdot \frac{U(\Delta h)}{2} .$$

For example, if ρ_f = 915 kg·m^{-3}, the uncertainty contribution will be:

$$u_8(A_p)/A_p = 9 \cdot 10^{-6} \cdot (p/\text{MPa})^{-1}.$$

B4.9 – *Head correction – density of the pressure-transmitting fluid*

The density of the pressure-transmitting medium ρ_f has an effect only if the height difference between the two cross-floated pressure balances is not equal to zero. If the relative expanded uncertainty of ρ_f is $U_{rel}(\rho_f)$ = 2%, expressed at k = 2, the uncertainty contribution will be:

$$\frac{u_9(A_p)}{A_p} = \frac{u_{\rho_f}(A_p)}{A_p} = \frac{g \cdot \Delta h}{p_r + (\rho_f - \rho_a)\cdot g \cdot \Delta h} \cdot \frac{U_{rel}(\rho_f)\cdot \rho_f}{2} \approx \frac{g \cdot \Delta h}{p} \cdot \frac{U_{rel}(\rho_f)\cdot \rho_f}{2} .$$

For example, if ρ_f = 915 kg·m^{-3} and Δh = 5 cm, the uncertainty contribution will be:

$$u_9(A_p)/A_p = 4.5 \cdot 10^{-6} \cdot (p/\text{MPa})^{-1}.$$

B4.10 – *Surface tension of the pressure-transmitting fluid*

If the relative expanded uncertainty of the surface tension σ is $U_{rel}(\sigma)$ = 10%, expressed at k = 2, the uncertainty contribution will be:

$$\frac{u_{10}(A_p)}{A_p} = \frac{u_\sigma(A_p)}{A_p} = \frac{c}{\sum_i m_i \cdot g \cdot (1 - \rho_a / \rho_{m_i}) + \sigma \cdot c} \cdot \frac{U_{rel}(\sigma)\cdot \sigma}{2} \approx \frac{2}{p} \cdot \left(\frac{\pi}{A_0}\right)^{0.5} \cdot \frac{U_{rel}(\sigma)\cdot \sigma}{2} .$$

For example, if σ = 31.2 mN·m^{-1} and A_0 = 1.961004 mm^2, the uncertainty contribution will be:

$$u_{10}(A_p)/A_p = 4 \cdot 10^{-6} \cdot (p/\text{MPa})^{-1}.$$

The uncertainty of the piston circumference c, which is - in relative units - much smaller than $U_{rel}(\sigma)$, does not need to be considered.

B4.11 – *Tilt of the piston*

If the piston axis is not perfectly perpendicular, the force applied to the piston has to be corrected from the angle of tilt Θ:

$$F' = F \cdot \cos\Theta .$$

The distribution of force (or pressure) is non-symmetric. When experimentally treated in the correct way, this component is a small one. If $U(\Theta)$ presents the maximum uncertainty of the piston verticality, the tilt contribution will be:

$$\frac{u_{11}(A_p)}{A_p} = \frac{u_\Theta(A_p)}{A_p} = \sin\Theta \cdot \frac{U(\Theta)}{\sqrt{3}} .$$

The deviation from the vertical is generally checked by using a spirit level either built into the base of the pressure balance or put on the top of the piston. By this method it is usually possible to determine the tilt with $U(\Theta)$ = 5.8·10^{-4} rad. Herewith, the uncertainty contribution will be:

$u_{11}(A_p)/A_p = 2 \cdot 10^{-7}$.

B4.12 – *Discrimination threshold (cross-floating sensitivity)*

The discrimination is the pressure corresponding to the largest mass that produces no detectable change in the generated pressure. It may be taken into account when there is no reliable estimation of the repeatability of the pressure balance. In this example we assume that it is reflected in the repeatability of the measurements and, therefore, is presented in the type A uncertainty (component B3).

B5 Calculation of the combined uncertainty of the effective area

The type B relative standard uncertainty of A_p, $u_B(A_p)/A_p$ is calculated from the components of the type B uncertainty defined in B4 as:

$$\frac{u_B(A_p)}{A_p} = \left[\sum_{\substack{i=1 \\ i \neq 5}}^{11} \frac{u_i^2(A_p)}{A_p^2} - \frac{u_5^2(A_p)}{A_p^2} \right]^{0.5},$$

$u_B(A_p)/A_p = [(1.6 \cdot 10^{-5})^2 + (5.1 \cdot 10^{-5})^2 \cdot (p/\text{MPa})^{-2} + (1 \cdot 10^{-7})^2 \cdot (p/\text{MPa})^2]^{0.5}$.

The combined relative standard uncertainty of A_p, $u(A_p)/A_p$ is calculated from the type A and the type B uncertainty according to:

$$\frac{u(A_p)}{A_p} = \left[\frac{u_A^2(A_p)}{A_p^2} + \frac{u_B^2(A_p)}{A_p^2} \right]^{0.5}.$$

With the $u_A(A_p)/A_p$ defined in B3:

$u(A_p)/A_p = [(2.5 \cdot 10^{-5})^2 + (5.1 \cdot 10^{-5})^2 \cdot (p/\text{MPa})^{-2} + (1.2 \cdot 10^{-7})^2 \cdot (p/\text{MPa})^2 - 2 \cdot (1.0 \cdot 10^{-6})^2 \cdot (p/\text{MPa})]^{0.5}$.

The expanded uncertainty $U(A_p)/A_p$ is derived from the combined standard uncertainty by multiplying it by a coverage factor $k = 2$:

$$\frac{u(A_p)}{A_p} = 2 \cdot \frac{u(A_p)}{A_p},$$

$U(A_p)/A_p = [(5 \cdot 10^{-5})^2 + (1.0 \cdot 10^{-4})^2 \cdot (p/\text{MPa})^{-2} + (2.4 \cdot 10^{-7})^2 \cdot (p/\text{MPa})^2 - 2 \cdot (2.0 \cdot 10^{-6})^2 \cdot (p/\text{MPa})]^{0.5}$.

Appendix C

Example of uncertainty estimation for the pressure measured with a pressure balance

C1 Scope

This example presents the calculation of the uncertainty of the pressure measured with an oil-operated pressure balance. It is assumed that the effective area and its uncertainty have been evaluated for this pressure balance and are given as described in Appendix B. The pressure uncertainty estimation is based on the uncertainty of the effective area, the data included in the calibration certificates of the pressure balance and the environmental conditions. It should be taken into account that the uncertainty of the effective area already contains components of numerous uncertainty sources which were presented during the calibration of the pressure balance. Therefore, only additional uncertainty sources need to be considered here. To correctly define additional uncertainty sources, it is necessary to distinguish between two cases: 1) when all measurement and environmental conditions are the same as at the pressure balance calibration, and 2) when they are different. The second is a typical case when the pressure balance has been calibrated by laboratory A, which has issued a calibration certificate, and then is used for pressure measurement by laboratory B. In such a case, the measurement and environmental conditions should be considered as generally different. This case is analysed in the following.

C2 Definition of the pressure

The pressure p measured with an oil-operated pressure balance operated in gauge mode is given by the following expression:

$$p = \frac{\left[\sum_i m_i \cdot g \cdot (1 - \rho_a / \rho_{m_i}) + \sigma \cdot c\right] \cdot \cos(\Theta)}{A_p \cdot \left[1 + (\alpha_p + \alpha_c) \cdot (t - t_r)\right]} - (\rho_f - \rho_a) \cdot g \cdot \Delta h \,, \qquad \text{(C.1)}$$

where A_p is the effective area of the pressure balance and all other quantities as defined in Appendix B2.

C3 Type A standard uncertainty of the pressure

The type A uncertainty of the pressure is already included in the type A uncertainty of the effective area and does not need to be considered again. However, if the pressure balance has been calibrated more than one time, the stability of the calibrated quantities (effective area and masses) shall additionally be considered. If changes in the calibrated parameters are observed which exceed the uncertainties stated in the repeated calibrations, the variations should be included as additional uncertainty contributions in the uncertainty budget.

C4 Calculation of type B standard uncertainty of the pressure

The type B uncertainty estimation procedure for the pressure is the same as for the effective area (A3, B4). The uncertainty sources and their contributions to the pressure uncertainty are considered below.

C4.1 – *Effective area*

If $U(A_p)$ is the expanded uncertainty expressed using a coverage factor $k = 2$:

$$\frac{u_1(p)}{p} = \frac{u_{A_p}(p)}{A_p} \approx \frac{U(A_p)}{2 \cdot A_p} \,.$$

For example, if $U(A_p)$ is given as calculated in B5,

$$U(A_p)/A_p = [(5 \cdot 10^{-5})^2 + (1.0 \cdot 10^{-4})^2 \cdot (p/\text{MPa})^{-2} + (2.4 \cdot 10^{-7})^2 \cdot (p/\text{MPa})^2 - 2 \cdot (2.0 \cdot 10^{-6})^2 \cdot (p/\text{MPa})]^{0.5},$$

the uncertainty of pressure will be:

$$u_1(p)/p = [(2.5 \cdot 10^{-5})^2 + (5.0 \cdot 10^{-5})^2 \cdot (p/\text{MPa})^{-2} + (1.2 \cdot 10^{-7})^2 \cdot (p/\text{MPa})^2 - 2 \cdot (1.0 \cdot 10^{-6})^2 \cdot (p/\text{MPa})]^{0.5}.$$

C4.2 – *Mass*

If the pressure balance is operated with the same mass set with which it has been calibrated, the uncertainty of the masses does not need to be considered, because it is already included in the uncertainty of the effective area.

C4.3 – *Temperature of the piston-cylinder assembly*

The uncertainty of the temperature of the piston-cylinder assembly has already been included in the uncertainty of the effective area and does not need to be considered again, provided that the same thermometer is used as at the calibration of the pressure balance. If another temperature measuring system is used, and the temperature uncertainty during the calibration is not known, an additional consideration of the temperature effect may be necessary. In this example we assume that the thermometer used in the calibration and application of the pressure is the same.

C4.4 – *Thermal expansion coefficient of the piston-cylinder assembly*

The contribution of the thermal expansion coefficient of the piston and the cylinder depends on the deviation of the piston-cylinder assembly's temperature from the reference temperature. If the temperature deviation at the pressure balance used is not larger than at this calibration, no additional uncertainty component arises because the effect has already been included in the uncertainty of the effective area.

C4.5 – *Local gravity acceleration*

The uncertainty of the acceleration must be considered, because it has not been included into the uncertainty of the effective area. If the uncertainty of the gravity acceleration $U(g)$ is expressed at $k = 3$:

$$\frac{u_5(p)}{p} = \frac{u_g(p)}{p} \approx \frac{U(g)}{3 \cdot g}.$$

For example, if the gravity acceleration has been determined by calculation from the local longitude, latitude and altitude, its uncertainty is expressed at $k = 3$ is $U(g) = 4 \cdot 10^{-5} \times g$:

$$u_5(p)/p = 1.3 \cdot 10^{-5}.$$

C4.6 – *Air buoyancy – density of air*

In this example it is presumed that the atmospheric pressure, and the ambient temperature and humidity are not measured when applying the pressure balance, and, for calculating the buoyancy correction, the conventional value of the air density $\rho_{a0} = 1.2$ kg·m^{-3} is taken instead. The maximum air density variations in the laboratory have been demonstrated to be within ± 5%, $U(\rho_a) = 5 \cdot 10^{-2} \times \rho_a$, expressed at $k = 3$. With this,

$$\frac{u_6(p)}{p} = \frac{u_{\rho_a}(p)}{p} \approx \frac{\sum_i m_i/\rho_{m_i}}{\sum_i m_i} \cdot \frac{U(\rho_a)}{3}$$

results. For example, with the weights' density being the same for all the main weights and equal to $\rho_{mi} = 7920$ kg·m^{-3}, the uncertainty contribution will be:

$$u_6(p)/p = 2.5 \cdot 10^{-6}.$$

C4.7 – *Air buoyancy – density of weights*

If the pressure balance is operated with the same mass set with which it has been calibrated, the uncertainty of the weights' density does not need to be considered, because it is already included in the uncertainty of the effective area.

C4.8 – *Head correction – height difference*

In this example it is assumed that the height difference is controlled with an expanded uncertainty $U(\Delta h) = 4$ mm, expressed at $k = 2$, which is larger than at the pressure balance calibration. Then its contribution will be:

$$\frac{u_8(p)}{p} = \frac{u_{\Delta h}(p)}{p} \approx \frac{\rho_f \cdot g}{p} \cdot \frac{U(\Delta h)}{2}.$$

For example, if ρ_f = 915 kg·m^{-3}, the uncertainty contribution will be:

$$u_8(A_p)/A_p = 1.8 \cdot 10^{-5} \cdot (p/\text{MPa})^{-1}.$$

C4.9 – *Head correction – density of the pressure-transmitting fluid*

The density of the pressure-transmitting medium ρ_f has an effect only if the height difference between the two cross-floated pressure balances is not equal to zero. It is assumed here that the height difference does not exceed 5 cm, which has already been considered in the uncertainty budget of the effective area so that no additional uncertainty contribution is required.

C4.10 – *Surface tension of the pressure-transmitting fluid*

The surface tension uncertainty has already been considered as a contribution to the effective area uncertainty and therefore does not need to be analysed again.

C4.11 – *Tilt of the piston*

In this example it assumed that the piston tilt is measured with the same built-in spirit level as at the calibration. As its effect has already been included into the uncertainty of the effective area, no further consideration is required.

C4.12 – *Discrimination threshold (pressure sensitivity)*

The discrimination threshold is already included in the type A uncertainty of the effective area and does not need to be considered here.

C5 Calculation of the combined uncertainty of the pressure

The combined relative standard uncertainty of p, $u(p)/p$ is calculated from the uncertainty contributions defined in C4.1 to C4.12. In the considered example, it will be:

$$\frac{u(p)}{p} = \left[\frac{u_1^2(p)}{p^2} + \frac{u_5^2(p)}{p^2} + \frac{u_6^2(p)}{p^2} + \frac{u_8^2(p)}{p^2}\right]^{0.5},$$

$$u(p)/p = [(2.8 \cdot 10^{-5})^2 + (5.3 \cdot 10^{-5})^2 \cdot (p/\text{MPa})^{-2} + (1.2 \cdot 10^{-7})^2 \cdot (p/\text{MPa})^2 - 2 \cdot (1.0 \cdot 10^{-6})^2 \cdot (p/\text{MPa})]^{0.5}.$$

The expanded relative uncertainty $U(p)/p$ is derived from the combined standard uncertainty by multiplying it by a coverage factor k = 2:

$$\frac{u(p)}{p} = 2 \cdot \frac{u(p)}{p},$$

$$U(p)/p = [(5.6 \cdot 10^{-5})^2 + (1.1 \cdot 10^{-4})^2 \cdot (p/\text{MPa})^{-2} + (2.4 \cdot 10^{-7})^2 \cdot (p/\text{MPa})^2 - 2 \cdot (2.0 \cdot 10^{-6})^2 \cdot (p/\text{MPa})]^{0.5}.$$